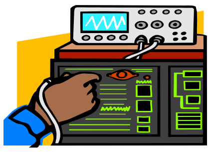
NOTRE DAME UNIVERSITY

FACULTY OF ENGINEERING

Department of Electrical, Computer & Communication Engineering

EEN 202



Circuits Analysis II

Solution* for HW 3

* Problem Solutions Extracted from the Solution Manual of:
"Basic Engineering Circuit Analysis", by J.D. Irwin and R. M. Nelms, 9th Edition, Wiley, 2008

Instructor: G. Hassoun

9.3 The voltage and current at the input of a circuit are given by the expressions

$$v(t) = 170 \cos(\omega t + 30^\circ) \text{ V}$$
$$i(t) = 5 \cos(\omega t + 45^\circ) \text{ A}$$

Determine the average power absorbed by the circuit.

SOLUTION:

$$V(t) = 170 \cos(\omega t + 30^\circ) V$$
$$i(t) = 5 \cos(\omega t + 45^\circ) A$$

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

$$P = \frac{170(5)}{2} \cos(30^\circ - 45^\circ)$$

$$P = 410.52 \text{ W}$$

9.7 Calculate the power absorbed by each element in the circuit in Fig. P9.7.

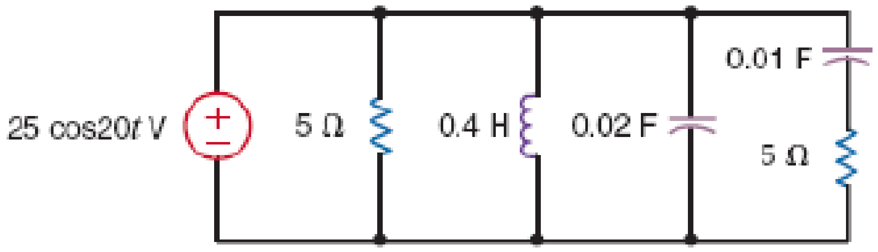


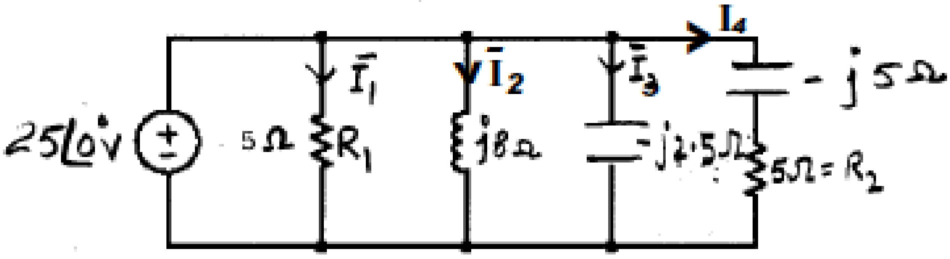
Figure P9.7

SOLUTION:

$$\bar{Z}_L = j\omega L = j(20)(0.4) = j8\,\Omega$$

$$\bar{Z}_{C_1} = \frac{1}{j\omega C_1} = \frac{1}{j(20)(0.02)} = -j2.5\,\Omega$$

$$\bar{Z}_{C_2} = \frac{1}{j\omega C_2} = \frac{1}{j(20)(0.01)} = -j5\,\Omega$$



$$\bar{I}_1 = \frac{25\angle 0^\circ}{5} = 5\angle 0^\circ\,A$$

$$\bar{I}_2 = \frac{25\angle 0^\circ}{j8} = 3.125\angle -90^\circ\,A$$

$$\bar{I}_3 = \frac{25\angle 0^\circ}{-j2.5} = 10\angle 90^\circ\,A$$

$$\bar{I}_4 = \frac{25\angle 0^\circ}{5 - j5} = 3.54\angle 45^\circ A$$

$$P_{R_1} = \frac{I_1^2 R}{2} = \frac{5^2(5)}{2}$$

$$P_{R_1} = 62.5\ W$$

$$P_{R_2} = \frac{I_4^2 R}{2} = \frac{(3.54)^2(5)}{2}$$

$$P_{R_2} = 31.33\ W$$

9.14 Given the network in Fig. P9.14, find **(a)** the power supplied and the average power absorbed by each element (**(b)** 1-Ω resistor, **(c)** 2-Ω resistor, **(d)** inductor, **(e)** capacitor).

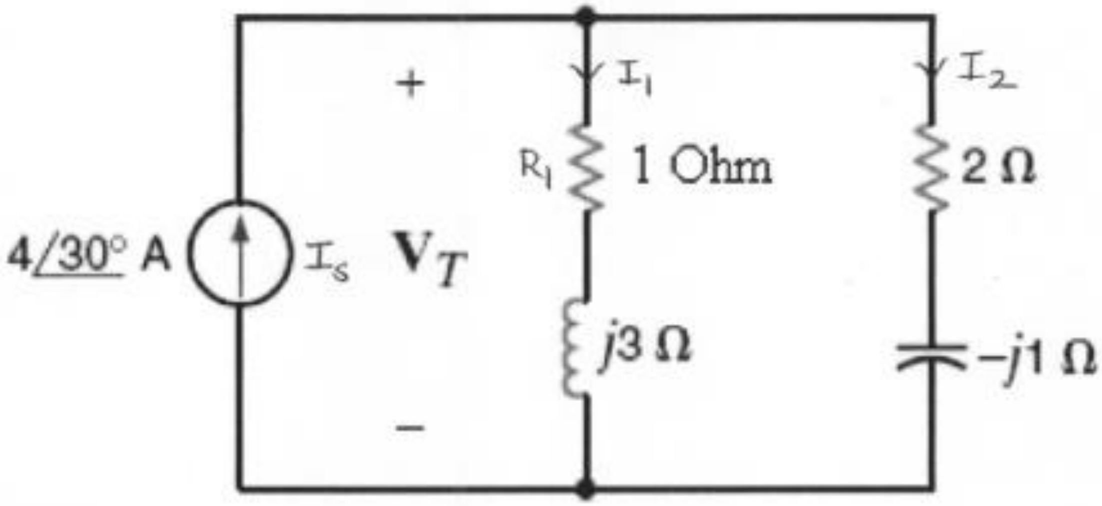


Figure P9.14

Find the power supplied at $t = 0$.

Solution: 9.14

(a) Let, $Z_1 = 1 + j3\ \Omega$ and $Z_2 = 2 - j1\ \Omega$

and $Z_T = \dfrac{Z_1 Z_2}{Z_1 + Z_2} = 1.961 \angle 11.31^\circ\ \Omega$

$V_T = 4\angle 30^\circ\, Z_T = 7.844 \angle 41.31^\circ$ V

For the source

$P_S(t) = \dfrac{4(7.844)}{2}\left[\cos(2\omega t + 71.31^\circ) + \cos 11.31^\circ\right]$

$= 15.688\left[\cos(2\omega t + 71.31^\circ) + 0.981\right]$

$P_S(0) = 15.39$ W

$$\boxed{P_S(0) = 15.4\ W}$$

(b) $I_1 = V_T / Z_1 = 2.481 \angle -30.26^\circ\ A$

$$P_{R_1} = \frac{I_{1M}^2}{2} R_1 = 3.078\ W$$

$$\boxed{P_{R_1} = 3.08\ W}$$

(c) $I_2 = V_T / Z_2 = 3.508 \angle 67.875^\circ\ A$

$$P_{R_2} = \frac{I_{2M}^2 R_2}{2} = 12.306\ W$$

$$\boxed{P_{R_2} = 12.3\ W}$$

(d) $\boxed{P_L = 0\ W}$

(e) $\boxed{P_C = 0\ W}$

9.16 Given the network in Fig. P9.16, show that the power supplied by the sources is equal to the power absorbed by the passive elements.

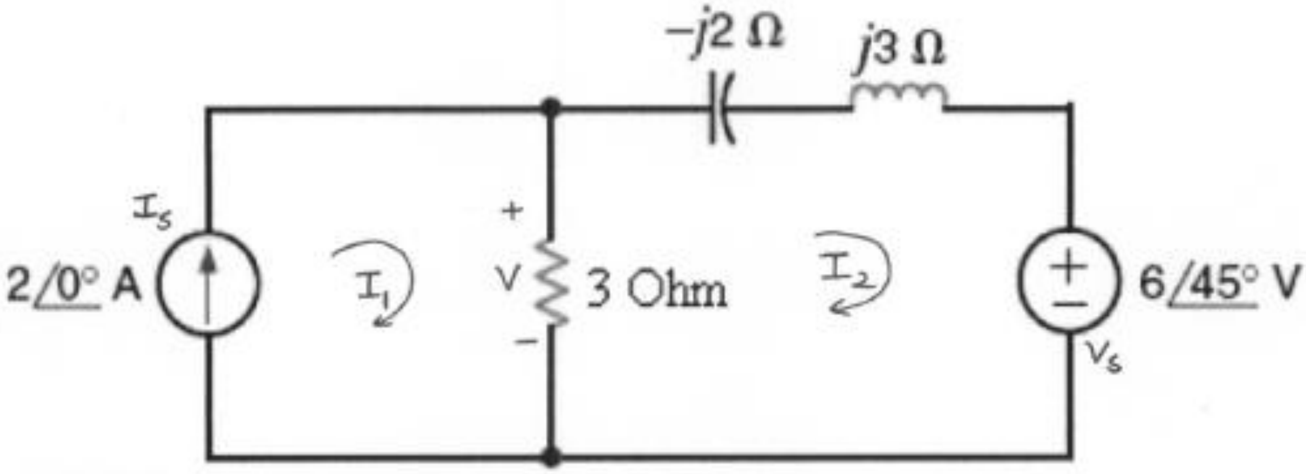


Figure P9.16

Find **(a)** power absorbed and **(b)** power supplied.

Solution: 9.16

$I_1 = 2\angle 0°\ A$ — ①

$-3I_1 + I_2(3+j1) = -6\angle 45°$ — ②

From equations ① and ②, we get

$$\begin{bmatrix} 1 & 0 \\ -3 & 3+j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2\angle 0° \\ -6\angle 45° \end{bmatrix}$$

$\Rightarrow I_2 = 1.453\angle -85.941°\ A$

$V = 3(I_1 - I_2) = 7.161\angle 37.374°\ V$

I_s supplies $P_{I_s} = \dfrac{I_{sM} V_M}{2} \cos(\theta_V - \theta_{Is})$

$\Rightarrow P_{I_s} = 5.691\ W$

V_s absorbs $P_{V_s} = \dfrac{V_{SM} I_{2M}}{2} \cos(\theta_{Vs} - \theta_{I2})$

$= -2.856\ W$

So, V_s actually delivers 2.856 W

$P_R = \frac{V_M^2}{2R} = 8.547\ W$

$P_L = P_C = 0\ W$

Power supplied $= 5.691 + 2.856 = 8.547\ W$

Power absorbed $= P_R = 8.547\ W$

$\therefore$ Power supplied = Power absorbed

(a) $\boxed{\text{Power supplied} = 8.55\ W}$

(b) $\boxed{\text{Power absorbed} = 8.55\ W}$

9.22 Determine the average power absorbed by a 2-Ω resistor connected at the output terminals of the network shown in Fig. P9.22.

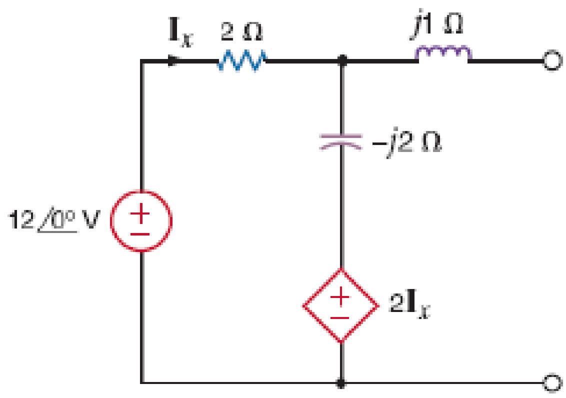
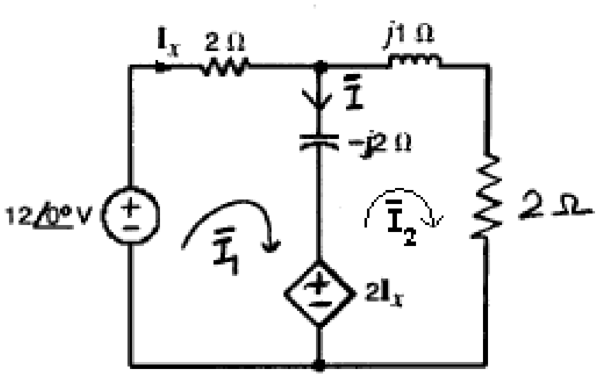


Figure P9.22

SOLUTION:



KCL: $\bar{I}_1 = \bar{I} + \bar{I}_2$ $\qquad\qquad$ $\bar{I} = \bar{I}_1 - \bar{I}_2$

KVL around left loop:

$2\bar{I}_1 - j2\bar{I} + 2\bar{I}_x = 12\angle 0^\circ$

$\bar{I}_x = \bar{I}_1$

$$2\bar{I}_1 - j2(\bar{I}_1 - \bar{I}_2) + 2\bar{I}_1 = 12\angle 0^\circ$$

$$(4 - j2)\bar{I}_1 + j2\bar{I}_2 = 12\angle 0^\circ$$

KVL around right loop:

$$j1\bar{I}_2 + 2\bar{I}_2 - j2(-\bar{I}) = 2\bar{I}_x$$

$$j1\bar{I}_2 + 2\bar{I}_2 + j2(\bar{I}_1 - \bar{I}_2) = 2\bar{I}_1$$

$$(-2 + j2)\bar{I}_1 + (2 - j1)\bar{I}_2 = 0$$

$$(4 - j2)\bar{I}_1 + j2\bar{I}_2 = 12\angle 0^\circ$$
$$(-2 + j2)\bar{I}_1 + (2 - j1)\bar{I}_2 = 0$$

$$\bar{I}_1 = 2.49\angle -4.76^\circ \text{ A}$$
$$\bar{I}_2 = 3.15\angle -23.2^\circ \text{A}$$

$$P_{2\Omega} = \frac{I_2^2 R}{2} = \frac{(3.15)^2(2)}{2} = 9.92\,W$$

9.30 Determine the impedance $\mathbf{Z}_L$ (**(a)** determine the real part, **(b)** determine the imaginary part) for maximum average power transfer and **(c)** the value of the maximum average power absorbed by the load in the network shown in Fig. P9.30.

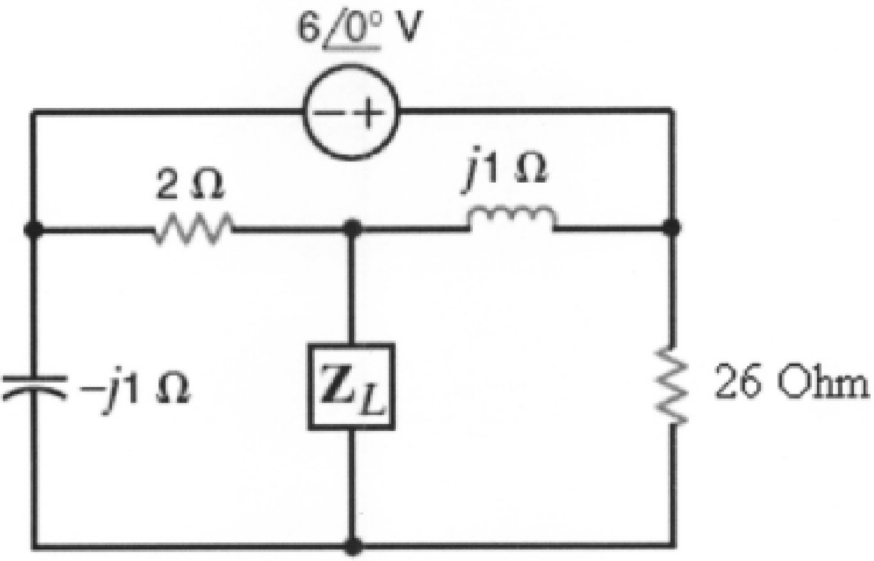
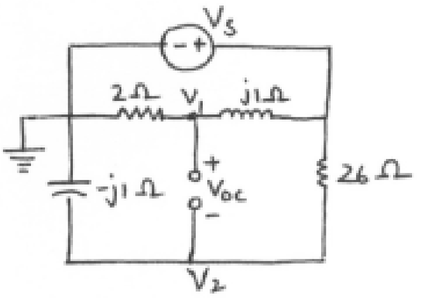


Figure P9.30

Solution: 9.30

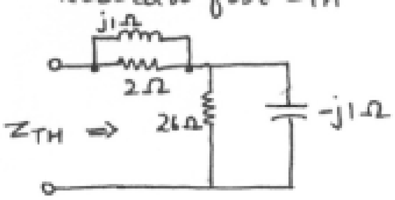


$V_1 = V_s \left[\frac{2}{2+j1}\right]$ $\quad V_2 = V_s\left[\frac{-j1}{26-j1}\right]$

$V_{oc} = V_1 - V_2$

$= 5.26 \angle -24.37^\circ$ V

Redraw for Z_{TH}



$Z_{TH} = \frac{2(j1)}{2+j1} + \frac{26(-j1)}{26-j1}$

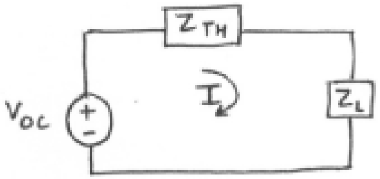
$= 0.438 - j\,0.198\ \Omega$

$= 0.48 \angle -24.85^\circ\ \Omega$

For maximum power transfer

$Z_L = Z_{TH}^* = 0.438 + 0.198j$

$I = \frac{V_{oc}}{Z_{TH}+Z_L} = 5.98 \angle -24.37^\circ$ A



$P_L = \frac{I_M^2}{2} R_L = 7.87$ W

(a) $\boxed{R_L = 0.438\,\Omega}$ (b) $\boxed{X_L = 0.198\,\Omega}$ (c) $\boxed{P_L = 7.87\text{ W}}$

9.33 In the network in Fig. P9.33, find $\mathbf{Z}_L$ for maximum average power transfer and the maximum average power transferred.

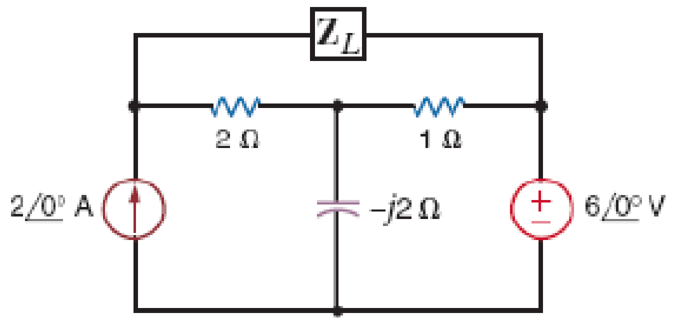
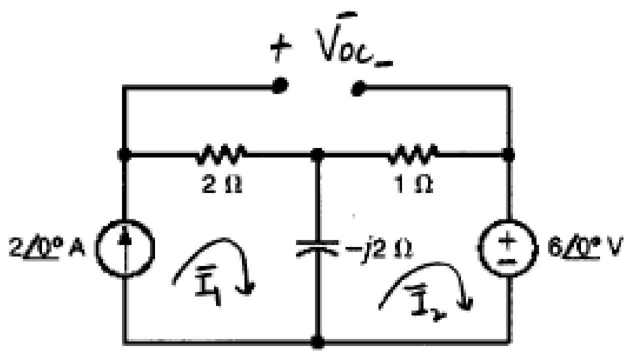


Figure P9.33

SOLUTION:



$$\text{KCL:} \quad \bar{I}_1 = \bar{I} + \bar{I}_2$$

$$\bar{I} = \bar{I}_1 - \bar{I}_2$$

$$\text{KVL right loop:} \quad 1(\bar{I}_2) + 6\angle 0^\circ - j2(-\bar{I}) = 0$$

$$\bar{I}_2 + 6\angle 0^\circ + j2(\bar{I}_1 - \bar{I}_2) = 0$$

$$j2\,\bar{I}_1 + (1 - j2)\,\bar{I}_2 = -6\angle 0^\circ$$

$$\bar{I}_1 = 2\angle 0^\circ \text{ A}$$

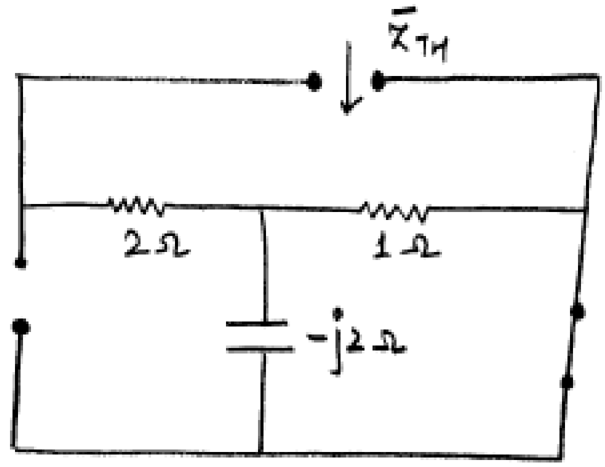
$$(1 - j2)\,\bar{I}_2 = -6\angle 0^\circ - j2(2\angle 0^\circ)$$

$$\bar{I}_2 = 3.22\angle -82.87^\circ \text{A}$$

$$V_{oc} = \bar{I}_1(2) + \bar{I}_2(1)$$

$$\bar{V}_{OC} = (2\angle 0^\circ)(2) + (3.22\angle -82.87^\circ)(1)$$

$$\bar{V}_{OC} = 5.44\angle -36^\circ \text{ V}$$

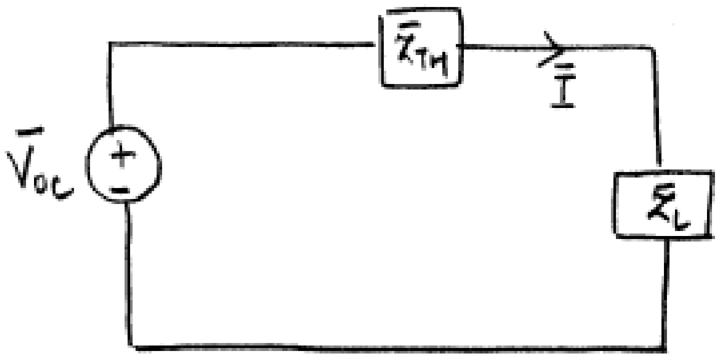


$$\bar{Z}_{TH} = 1 \| -j2 + 2$$

$$\bar{Z}_{TH} = \frac{1(-j2)}{1-j2} + 2 = 2.83\angle -8.13^\circ \Omega$$

$$\bar{Z}_{TH} = 2.8 - j0.4 \Omega$$

$$\bar{Z}_L = \bar{Z}_{TH}^* = 2.8 + j0.4 \Omega$$



$$\bar{I} = \frac{5.44 \angle -36^\circ}{2.8 - j0.4 + 2.8 + j0.4}$$

$$\bar{I} = 0.97 \angle -36^\circ \text{A}$$

$$P_{max} = \frac{I^2 R_L}{2} = \frac{(0.97)^2 (2.8)}{2}$$

$$P_{max} = 1.32 \text{ W}$$